Interpolation with Unequal Intervals

Divided differences:

Let the function $y = f(x)$ assumes the value $f(x_0), f(x_1), \ldots, f(x_n)$ corresponding to the arguments $x_0, x_1, \ldots, x_n$ respectively where the intervals $x_1 - x_0, x_2 - x_1, \ldots, x_n - x_{n-1}$ need not be equal.

Definitions:

The first divided difference of $f(x)$ for the arguments x_0, x_1 is defined as $\frac{f(x_1) - f(x_0)}{x_1 - x_0}$. It is denoted by $f(x_0, x_1)$ or $[x_0, x_1]$ or $\underset{x_1}{\Delta} f(x_0)$.

In otherwords,

$$f(x_0, x_1) = [x_0, x_1] = \underset{x_1}{\Delta} f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \rightarrow (1)$$

In the same notation, we have

$$f(x_1, x_2) = \underset{x_2}{\Delta} f(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$ and

$$f(x_{n-1}, x_n) = \underset{x_n}{\Delta} f(x_{n-1}) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}, \quad n = 1, 2, \ldots, n$$

The second divided differences of $f(x)$ for the three arguments x_0, x_1, x_2 is defined as

$$f(x_0, x_1, x_2) = \underset{x_1, x_2}{\Delta^2} f(x_0) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0} \rightarrow (2)$$

This shows that to find a second divided difference, we require three continuous arguments.

In the same way, we define the third divided differences of $f(x)$ for the four arguments x_0, x_1, x_2, x_3 as

$$\Delta^3_{x_1,x_2,x_3} f(x_0) = f(x_0,x_1,x_2,x_3) = \frac{f(x_1,x_2,x_3) - f(x_0,x_1,x_2)}{x_3 - x_0} \quad \rightarrow (3)$$

Equations (1), (2), (3) refer to divided differences of order one, two and three respectively.

Divided difference table :

Arguments x	Entry $f(x)$	1st div. diff $\Delta f(x)$	2nd div. diff $\Delta^2 f(x)$	3rd div. diff $\Delta^3 f(x)$
x_0	$f(x_0)$	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	$f(x_0,x_1,x_2,x_3)$
x_1	$f(x_1)$	$f(x_1,x_2)$	$f(x_1,x_2,x_3)$	$f(x_1,x_2,x_3,x_4)$
x_2	$f(x_2)$	$f(x_2,x_3)$	$f(x_2,x_3,x_4)$	
x_3	$f(x_3)$	$f(x_3,x_4)$		
x_4	$f(x_4)$			

Ex:1 Form the divided difference table for the following data.

x :	-2	0	3	5	7	8
$y = f(x)$:	-792	108	-72	48	-144	-252

Sol

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-2	-792	$\frac{108-(-792)}{0-(-2)} = 450$	$\frac{-60-450}{3-(-2)} = -102$	$\frac{24+102}{5+2} = 18$	$\frac{-9-18}{7+2} = -3$
0	108	$= -60$	$= 24$	$= -9$	
3	-72				
5	48				
7	-144				
8	-252				